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OPTIMIZATION OF DIRECT FIRE EFFECTS BY AIMPOINT SELECTION PART I. BASIC THEORY

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This study discusses the factors related to the optimal selection of aimpoints for direct fire. Mean point of impact or bias errors and round-to-round or precision errors are considered. The concepts and techniques related to the problem are developed through a series of examples. A study looking into the effects of varying the precision errors is presented.

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I. INTRODUCTION

This paper investigates several methods of allocating a fixed number of rounds to optimize the probability that a direct-fire, heliborn gun will hit another helicopter. These methods are more succinctly called "strategies," or "policies."

In order to hit the target, many sources of error need to be controlled. (The word "error," as used here, is the distance between the actual point of impact and the intended point of impact. It does not mean simply a "mistake" on the part of the gunner, although a mistake by the gunner would be *a source* of error.) The distance between the two aircraft must be estimated, and with that distance the time-of-flight for a round is computed. From the time-of-flight and an estimate of the target's relative velocity (speed and direction), the target's location at impact is then predicted. Wind speed measurements are used to adjust the offset angle, at which to aim the gun.

This paper examines the sources of three broad types of errors: mean-point-of-impact (MPI) errors, round-to-round errors, and errors due to vibration.

1. MPI errors. These errors have a high correlation across shots, i.e., roughly the same error is found among all shots when the gun is aimed at a given point. Examples are an improper alignment of the sighting system, and a mistake in estimating the wind speed. When the MPI error is less than the size of the target, the pattern of shots will cluster around a point some distance from the intended point of impact (e.g., the target center). When the MPI error exceeds the size of the target, the center of the pattern will simply miss the target; thus, it is possible that aiming directly at the target will guarantee a miss. MPI errors are also called "bias" errors.

2. Round-to-round errors. Examples arise from the differences in the ammunition from round to round, and from variations in the wind speed from round to round. Such errors are also called "precision" or "dispersion" errors.

3. Vibrational errors. The oscillations of the gun-barrel produced by vibrations in the aircraft gives rise to these errors. The oscillations are the result of the gun blasts' reverberations throughout the aircraft, the effects of the rotor blade (for a helicopter), and other sources of vibratory mechanical energy.

It is the thesis of this paper that, through a judicious selection of "aimpoints," it is possible to maximize the probability of hitting the target, for a given number of rounds. The gunner (or more generally, the fire-control system) is responsible for selecting the aimpoints.

The geometry used is as follows:

1. Consider the line connecting the gun of the firing helicopter to the predicted location of the target helicopter.

2. Consider the plane perpendicular to the line described above in (1.), intersecting the predicted target location.

3. The points in this plane will be referred to as aimpoints. The origin is the predicted target location.

4. An aimpoint is uniquely defined by its deviation, in azimuth and elevation, from the origin.

This paper discusses a method that generates an optimal set of aimpoints for a fixed number of rounds, and then considers the problems caused by vibrational errors. Starting with a simple case and progressively generalizing the results, it leads to a theoretically optimized solution. This solution denotes the upper bound in performance for any firing policy, or strategy. This paper concludes with a comparison and evaluation of several alternative firing strategies.

II. BACKGROUND

The delivery errors of various weapons have been successfully modeled as bivariate, normal, and random variables.

- In 1941 Detrick, Kent, and Smith (1) published a paper describing the optimal spacing of bombs. In that paper they developed the idea of maximizing the probability of hitting the target at least once. They confined their discussion to the one-dimensional case.

- In 1945 Kolmogorov, Svesnikov, and Gubler (2) published a collection of articles on the selection of aimpoints for the one-, two-, and three-dimensional cases. In these papers "artificial dispersion," caused by using different aimpoints or by increasing the round-to-round firing error, is briefly examined for its potential to increase the amount of target damage. The damage caused by several shots fired at different aimpoints can be calculated based on (a) damage function, (b) MPI errors, (c) precision errors, and (d) aimpoints. Chapter Twenty of Army Weapons System Analysis (3) mentions several difficulties associated with this type of problem and suggests that the ultimate solutions must be sought case-by-case.

- Kisi (4) discussed the idea of increasing artificial dispersion by increasing the precision error.

- Fendrikov and Yakolev (5) mentioned an aimpoint strategy for indirect artillery fire.

- Sandmeyer (6,7) found the optimal aimpoint policy for indirect artillery fire. He showed his method is superior to both the method of Fendrikov and Yakolev and the method used by the Battery Computer System (currently in use by the U.S. artillery); and he established a theoretical upper bound for the optimal aimpoint policy. This paper follows the method of Sandmeyer.

Sandmeyer (6) applies a method previously applied to the problem of optimal search to the selection of optimal aimpoints. A discussion of the theory in the context of that problem follows. The basic task of optimal search is illustrated by the following: Suppose someone has lost a thimble and has only a fixed amount of time to look for it. How should this time be spent in order to maximize the probability of finding the thimble? Obviously the search time will be more efficiently spent if the searcher can approximate the likelihood that it is in the kitchen, the dining room, or the utility closet, for example. This concept is more formally treated by the use of the notions of the "Target-Location Density," or TLD, and the "detection function," or DF.

- The TLD is a function that quantifies the probability that a target will be within a specific incremental area. The integral of this function over an area gives the probability that the target is in that area.

- The DF is the probability that a searcher, looking at an incremental area, will detect a target located there given that the target is indeed there. Thus the DF measures the efficiency of the searcher. (When the DF is constant, it means that the searcher is no bettei an observer -- and no worse -- when he is looking for the thimble in the dining room than he is when looking in the utility closet, for example. Again, when the DF is 1/2, then the searcher has a fifty-fifty chance of spotting the thimble, when he is looking in the room where the thimble actually is).

Koopman (8, 9, 10) discusses these issues in detail and relates some of his experiences in a Naval Operations Research group during World War II. Stone (11) discusses methods of search that are optimized over "time." Koopman (12) uses a theorem of Gibbs (13) to describe the theoretical optimal allocation of "effort" for a search. Koopman's technique divides the TLD into two regions: a region to be searched, and a region to be neglected. In the region to be searched, the amount of effort applied to each incremental area is functionally proportional to the TLD in that area. When the search is done, if the target has not been found, then the resulting modified TLD will be flat, or constant, in the area that was searched and will be lower (or equal) in the neglected region. Figure 1 illustrates this process. In Figure 1a the original TLD is shown. Figure 1b shows the a posteriori TLD.

For a constant-valued detection function, this technique amounts to finding the height of a plane, parallel to the X-Y plane, such that the mass of the TLD above the plane "corresponds to" the total search-effort available (this plane will be referred to as the cut-off plane). This surface above the upper plane can be called the "optimal-effort surface." The projection of this mass onto the X-Y plane (i.e., the removal of the mass between the planes) will indicate the preferred areas to be searched. The amount of search in each incremental area is proportional to the TLD above that area. The effect of an optimal search, as said above, is a flattening of the original TLD. The optimal-effort surface can be difficult to find for many TLDs, including most nonexponential detection functions. The details associated with satisfying the requirements of the technique are contained in the examples given below.

The "optimal search" techniques can be directly applied to the "optimal aimpoint" problem. The meaning of the TLD does not change. The meaning of the detection function is changed from the probability of finding the target, given that it is in the region being searched, to the probability of hitting the target, given that it is in the region being shot at.

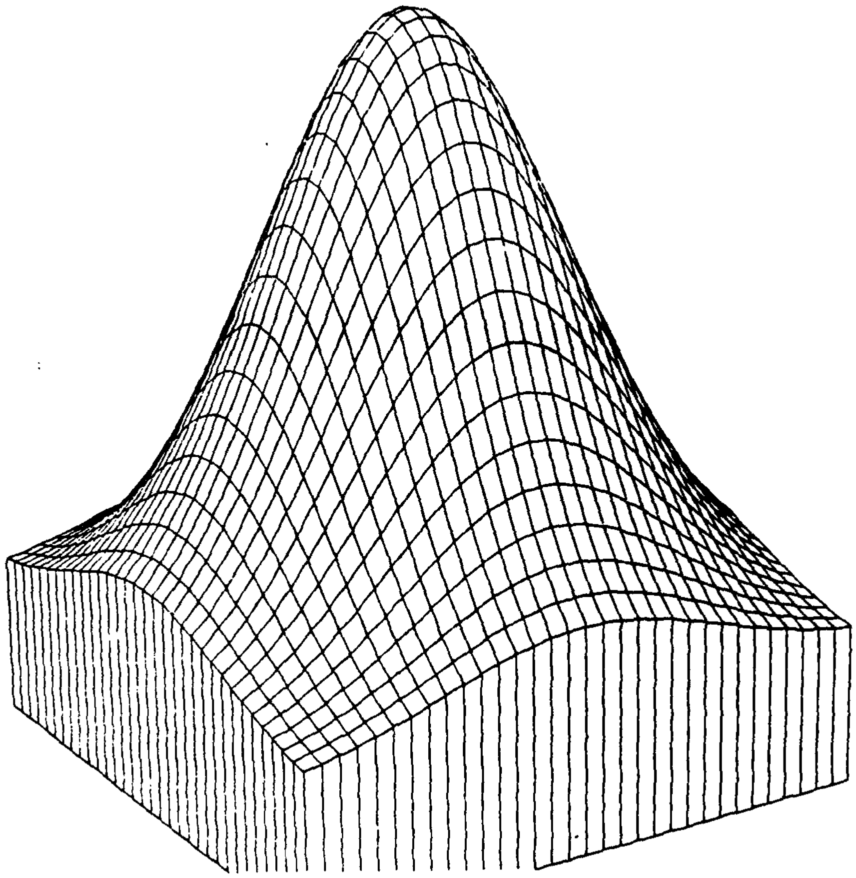

Figure 1a. Target location density before shooting.

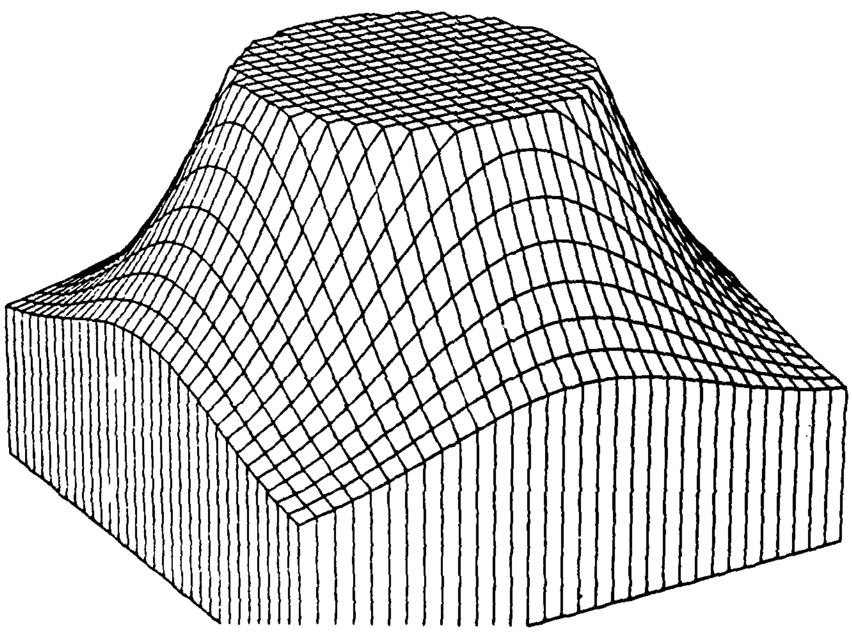

Figure 1b. Target location density after shooting.

The meaning of the "amount of TLD above the two parallel planes" is changed from the general "amount of effort available for the search" to the "amount of ammunition available for the gun." The "optimal-ammunition surface" is again found by using Koopman's application of Gibbs' theorem. (This procedure could likewise be applied to probability-of-kill problems: simply replace "hit" with "kill" in the foregoing terms.)

To the optimal-aimpoint problem, Sandmeyer added the influence of round-to-round and MPI errors. The work of Stone can be applied so that P{hit} is always maximized for the number of rounds fired.

III. DISCRETE CASE

Example 1.

As an illustration of the above ideas, three methods for solving a single problem will be discussed. Consider a 3x3 matrix, where the value in cell (i,j) [i and j running from 1 to 3] represents the probability that the target is in cell (i,j).

i=1	0.20	0.10	0.05
=2	0.10	0.30	0.05
=3	0.05	0.10	0.05
	j=1	=2	=3

The probability that the target is located in the region corresponding to cell (2,2) is 0.3. Assume there is a fifty-percent chance of hitting the target if a shot is fired into the cell. (That is, assume that the "detection function" is 1/2 for every cell). We assume zero error in delivery: either the weapon system is perfect in its ability to hit that cell or the area of the cell is large compared to the delivery error. What sort of firing policy will maximize the probability of hit for twenty rounds?

1. Method One: Proceed sequentially using a maximum-likelihood method; fire each shot at the area with the highest TLD. After each shot, replace the ·ell's TLD with the probability that the target is still there but unhit. That is, replace the original TLD [call it TLDo] with "TLDo (1-DF)," or, here, (0.3 x 1/2 =) 0.15. (That is, some of the probability is moved to the category of target hit.) The effects of a shot into a particular cell can be thought of in two ways either as transferring probability from a particular cell to the category of target hit or as a reduction in the probability that the target remains unhit in the cell. This paper uses the latter interpretation. For the allocation of the twenty rounds, the procedure would follow this pattern:

Shot 1: Choose cell (2,2), since it contains the greatest TLD.
Replace 0.3 with (0.5 x 0.3 =) 0.15.

Shot 2: Choose cell (1,1), since it now contains the greatest TLD.
Replace 0.2 with (0.5 x 0.2 =) 0.1.

Shot 3: Choose cell (2,2), since it now contains the greatest TLD.
Replace 0.15 with (0.5 x 0.15 =) 0.075

Now several cells have the equal maximum probability of 0.10. Here, the gunner can randomly choose any one of those cells. (When this possibility exists there may be no unique optimal solution. If the gunner runs out of ammunition before all these maximums are attacked, one can only speak of "an" optimal solution.)

Method One not only indicates the number of shots to be delivered to each cell but also indicates the best sequence for delivery. This straightforward method can be used when one has a discrete TLD with "zero" delivery-error.

2. Method Two. This method introduces the techniques used when one has a continuous TLD. The method is to find the amount of ammunition to apply to the highest-valued cell to reduce it to the level of the cell containing the next lower value. Then apply the ammunition to both of those cells until level of the next lower cell is reached. In this continuous-TLD situation, we assume that the ammunition effort is also continuous (i.e., that the ammunition can be fired in fractional amounts). This assumption is necessary in order to calculate the optimal-ammunition surface. As an approximate "practical" interpretation of the assumption, one could consider applying "bursts" rather than single rounds to a given area, where the length of the burst is adjustable, and a shorter burst conserves ammunition that can be used in later bursts. The infantryman has an apt expression for this concept when he "hoses down" an area.

If a full burst were fired at cell (2,2), then the probability that the target was there but was not hit is 0.15. Since this value is lower than 0.2, the gunner has gone too far. The proper amount of ammunition to have fired at cell (2,2) was a fractional amount, enough to have reduced to TLD to 0.2 and no lower. Thus:

Burst 1: Reduce the TLD of cell (2,2) to the TLD of cell (1,1)

$$0.2 = 0.3{*}0.5{*}{*}n \rightarrow n = 0.585.$$

As a realistic interpretation, if cell (2,2) were an acre lot with the target somewhere within it, and a full burst were 1,000 bullets, it would be optimal for the gunner to fire 585 bullets into the acre and no more, until he began engaging this cell in conjunction with another cell.

Note here, that the DF of 1/2 is based on the presumption of the full burst: If the gunner fires 1,000 rounds into the acre lot, then the chances of hitting the target in it are 1/2. Reasonably, then, if he fires only 585 rounds into the area, his chances of hitting the target are less than 1/2. In effect, the use of the fractional burst is a way of altering the DF.

(Observe, too, that this exercise is applying the technique for a continuous TLD to a situation in which the TLD is actually discrete.)

Burst 2: Find the amount of effort that will reduce 0.2 to 0.1

$$0.1 = 0.2*0.5**n \rightarrow n = 1$$

Burst 3: Find the amount of effort that will reduce 0.1 to 0.05

$$0.05 = 0.1*0.5**n \rightarrow n = 1$$

The derivation thus far has not made use of the fact that the gunner's ammunition is limited. He has so many bursts and not more. In this continuous case, the optimal allocation of ammunition can now be derived by means of an "Effort Matrix" expressed in terms of "E" -- the "minimal-effort value." The value in each cell represents the effort to be applied to the corresponding area.

For the Effort Matrix corresponding to the given problem, each of the cells with TLDs of 0.05 has some undetermined amount of effort E in it. The cells corresponding to the TLD cells with entries of 0.1 have one full unit more of effort in them, so they carry the value 1+E. The cell with the TLD value of 0.2 has yet one more unit of effort, so its entry is 2+E. Finally, the cell with a value of 0.3 has an additional 0.58 units of effort (rounded to two decimal places). Its entry is 2.58+E.

2 + E	1 + E	E
1 + E	2.58 + E	E
E	1 + E	E

The total effort expended is to be equal to twenty "bursts" (continuing the analogy that one burst is 1,000 rounds, we could assume the gunner has 20,000 rounds available), so by summing the cells of the matrix and setting the total equal to twenty bursts, we can find the optimal effort for each cell:

$$20 = 9E + 7.58 \quad \text{(Equivalently: } 20{,}000 = 9E + 7{,}580$$
$$E = 1.38 \text{ bursts} \quad E = 1{,}380 \text{ bullets)}$$

Thus, where the length of the burst is variable, the optimal solution is:

3,380	2,380	1,380
2,380	3,960	1,380
1,380	2,380	1,380

It will be seen that the total number of bullets expended is 20,000 and, of course, that the allocation has not been in 1000-round bursts.

Returning to the original case of 20 rounds, if the ammunition could not be subdivided, it would be necessary to enter integer numbers of rounds in each cell. As an approximation, we would round off the values in the cells to integer amounts and make further adjustments to ensure that the sum is twenty. One plausible solution in this case is

3	3	1
3	4	1
1	3	1

3. Method Three. This method uses the ideas of Koopman; Gibbs (1928) originally applied these ideas to a physics problem. Koopman derives two formulas that can be used to find the optimal munition surface. The first formula is used to find the area to shoot at. After the area to shoot at has been defined, the second formula is used to determine the amount of munition to apply to each point. This method is valid when the TLD is continuous and is a formalization of the technique used in Method 2. The equations used are:

$$\Phi = \iint_A \left[(\ln (p(x,y)\, w(x,y)) - \ln \lambda)/w(x,y)\right] dxdy \qquad (1)$$

and

$$\phi(x,y) = \frac{1}{w(x,y)} \ln \left(\frac{p(x,y)\, w(x,y)}{\lambda} \right) \qquad (2)$$

Φ is the total munition effort

$\phi\,(x,y)$ is the munition density at x,y

p(x,y) is the probability of the target being at x,y

λ is the height of the cut off plane

A is the fire zone

$e^{-w(x,y)}$ is the probability of missing a target located at (x,y).

Equation 1 divides the TLD into the two areas based on the number of munitions available. Equation 2 is used to determine the amount of munition to apply to each point. As applied to the current example the steps are as follows:

First, express the probability of a miss as an exponential. As we fire more shots into a specific region the returns on each shot (probability of hit) diminish in proportion to the probability that the target is unhit in that region. This diminishing rate of return is captured by the exponential function.

Second, solve Equation 1 for λ. λ is the height of the plane that cuts the TLD at the level appropriate for that amount of effort. (Note that integration can be replaced by summation for this discrete case).

Third, find the amount of effort at each of the nine points using Equation 2.

Fourth, find an integer solution.

Implementing these steps yields the following:

$$e^{-w(x,y)} = p_k \rightarrow w(x,y) = -\ln .5 = .6931$$

Note that w(x,y) is constant and can be replaced by w.

$$20 = \sum_{i=1}^{9} (\ln (p_i * w) - \ln \lambda)/w$$

$$\ln \lambda = \frac{20w - \sum \ln p_i w}{-9} \rightarrow \lambda = .0133$$

Using $\phi_i = \frac{1}{w} \ln \left(\frac{p_i * w}{\lambda} \right)$ we get the optimal munition matrix

3.3819	2.3818	1.3817
2.3818	3.9669	1.3817
1.3817	2.3818	1.3817

Integerization of this solution yields the same result as the previous example. Note that this method will work for a continuous TLD.

4. Optimal Probability of Hit

To find the p(hit | 20 rounds and optimal) for all twenty rounds first observe that the probability mass of the original distribution left in each cell is the same, i.e.,

For cell (2,2)	$.3 * .5^{3.9669}$	= .01919
For cell (1,1)	$.2 * .5^{3.3819}$	= .01919
For the cells containing .1	$.1 * .5^{2.3818}$	= .01919
For the cells containing .05	$.05 * .5^{1.3817}$	= .01919

Visualize a plane cutting through the original distribution at .01919. The probability the target was hit is the sum of the density above this plane, or the complement of the mass under the plane and is equal to 1-.01919 x 9=.82729.

This optimal value for a probability of a hit is physically unrealizable since munitions cannot be fired in fractional amounts. If we assume the best approximation to the optimal munition density is expressed by the following matrix:

$$\begin{matrix} 3 & 3 & 1 \\ 3 & 4 & 1 \\ 1 & 3 & 1 \end{matrix} ,$$

then we can find the probability of hitting by adding the probabilities of missing the target in each cell together and subtracting the result from 1.

For cell (1,1)	$.2 * .5^3$	$= .025$
For cell (1,2)	$.1 * .5^3$	$= .0125$
For cell (1,3)	$.05 * .5^1$	$= .025$
For cell (2,1)	$.1 * .5^3$	$= .0125$
For cell (2,2)	$.3 * .5^4$	$= .01875$
For cell (2,3)	$.05 * .5^1$	$= .025$
For cell (3,1)	$.05 * .5^1$	$= .025$
For cell (3,2)	$.1 * .5^3$	$= .0125$
For cell (3,3)	$.05 * .5^1$	$= .025$

Then the probability of hitting the target would be .81875.

5. Overkill

Overkill occurs when the fire control system or gunner fires excessively into one section of the TLD. The amount of overkill in a particular section would have been more profitably applied to another area of the TLD. As an example of overkill consider the probability of hit if all twenty rounds are fired into cell (2,2). The probability of the target surviving in cell (2,2) would be 2.86×10^{-7}. The total probability of hit would be approximately .3. The aiming policy of shooting at the center of the TLD would reduce the probability of a hit from its optimal value of .82 to .3.

This concludes our look at the discrete case.

IV. CONTINUOUS CASE

In the continuous case Koopman's result guides us in finding the value for the cutoff plane for a given amount of effort. In using this equation note that the munition surface must be greater than zero at all points; we cannot fire negative amounts of munition at an unlikely area and counterbalance this by applying more munition to more probable area. The value $z = \ln(\lambda)$ is the cutoff plane of the surface $\ln(p(x,y)\, w(x,y))$. $\ln(\lambda)$ and the total amount of munition expended vary inversely. All regions where $\ln(p(x,y)\, w) < \ln(\lambda)$ are ignored. Assuming $w(x,y) = w$ and $\ln(\lambda)$ can be expressed as $\ln(p(x',y')\, w)$ Equation 1 can be expressed as

$$\Phi = \iint_A \Big[\ln(p(x,y)\, w) - \ln(p(x',y')\, w)\Big] dxdy/w. \tag{3}$$

This can be rewritten as

$$\begin{aligned} w\,\Phi &= \iint_A \Big[\ln(p(x,y)) + \ln w - \ln(p(x',y')) - \ln w\Big] dxdy \\ &= \iint_A (\ln p(x,y) - \ln p(x',y'))dxdy \end{aligned} \tag{4}$$

In this situation Equation 4 shows $\ln \lambda$ is inversely related to w. Equation 2 can be written as

$$\phi(x,y) = \frac{1}{w}\,[\ln p(x,y) - \ln p(x',y')]. \tag{5}$$

This indicates the amount of munition applied to each point is proportional to the difference between the TLD and the cutoff plane. These ideas are applied to the following problem.

Example 2:

Given a circular normal TLD and constant hit function, describe the optimal munition surface. The TLD is described by

$$p(x,y) = (2\pi\sigma^2)^{-1} e^{\frac{-(x^2+y^2)}{2\sigma^2}}.$$

Taking the natural log we have

$$\ln p(x,y) = \ln(2\pi\sigma^2)^{-1} + \frac{-(x^2+y^2)}{2\sigma^2}.$$

Rewriting Equation 1 for this problem we have

$$\Phi = \iint_A \left[\ln(2\pi\sigma^2)^{-1} + \frac{-(x^2+y^2)}{2\sigma^2} + \ln w - \ln\lambda\right] dxdy/w.$$

The integrand is the equation of a concave downward parabaloid.

Next change to polar coordinates

$$w\Phi = \int_0^{2\pi}\int_0^{A} \left[\ln(2\pi\sigma^2)^{-1} + \frac{-R^2}{2\sigma^2} + \ln w - \ln\lambda\right] RdRd\Theta. \tag{6}$$

The value in brackets as previously mentioned must be greater or equal to zero. In terms of A we will use the following expression for $\ln\lambda$

$$\ln\lambda = \ln(2\pi\sigma^2)^{-1} + \frac{-A^2}{2\sigma^2} + \ln w.$$

Equation 6 can be written

$$w\Phi = \int_0^{2\pi}\int_0^{A} \left[\ln(2\pi\sigma^2)^{-1} + \frac{-R^2}{2\sigma^2} + \ln w - \ln(2\pi\sigma^2)^{-1} - \frac{-A^2}{2\sigma^2} - \ln w\right] RdRd\Theta$$

$$= \int_0^{2\pi}\int_0^{A} \left[\frac{-R^3}{2\sigma^2} + \frac{A^2 R}{2\sigma^2}\right] dRd\Theta$$

$$= \int_0^{2\pi} \left(\frac{-A^4}{8\sigma^2} + \frac{A^4}{4\sigma^2}\right) d\Theta$$

$$= \frac{2\pi A^4}{8\sigma^2} = \frac{\pi A^4}{4\sigma^2} \rightarrow A^4 = \frac{4 w\Phi\sigma^2}{\pi}$$

$$\rightarrow \lambda = (2\pi\sigma^2)^{-1} e^{\frac{-1}{\sigma}\left(\frac{w\Phi}{\pi}\right)^{1/2}} w.$$

The optimal munition surface will be zero outside the circle of radius A; within the circle the munition density is given by Equation 2 which simplifies to

$$(A^2 - R^2)/(w\,2\sigma^2)$$

The probability of a hit for this example can be found by the following method. Note from Figure 1 that P(hit) is the volume bound by the curve p(R,Θ) and the cutoff plane z = λ or the difference between Figure 1a and 1b. The volume under the circular normal distribution is given by

$$1 - e^{\frac{-R^2}{2\sigma^2}}$$

This volume contains a cylinder of radius A and height p(A); so we must remove this volume from the previous value. The volume of the cylinder is

$$\pi A^2 h = 2\pi\sigma\left(\frac{\Phi w}{\pi}\right)^{1/2} \frac{1}{2\pi\sigma^2} e^{\frac{-1}{\sigma}\left(\frac{\Phi w}{\pi}\right)^{1/2}}$$

so the expression for p(hit) is

$$1 - \left(1 + \frac{1}{\sigma}\left(\frac{\Phi w}{\pi}\right)^{1/2}\right) e^{\frac{-1}{\sigma}\left(\frac{\Phi w}{\pi}\right)^{1/2}} \tag{7}$$

Next we extend this problem to quantify the relationship between intelligence or improved target location knowledge and firing more rounds. Assume a circular normal target location error with sigma of 100m. Let w = .5 (the probability of missing the target is $e^{-.5}$ or .61) and suppose there are ten rounds, each round having an effective radius of thirty meters. Notice that both the standard deviation of the TLD and the total amount of munition effort need to be in the same units; thus the total effort needs to be an area. For this example the total effort available (Φ) is the number of rounds multiplied by the area each round destroys or $10 * \pi * 30^2$. From Equation 7 above the probability of hit is

$$1 - \left(1 + \frac{1}{100}\left(\frac{10{*}.5{*}900{*}\pi}{\pi}\right)^{1/2}\right) e^{\frac{-1}{100}(4500)^{1/2}} = .146$$

If we instead fire 20 rounds, the probability of hit is .246; however, if we had reduced the target location error by fifty percent so sigma was 50m for 10 shots the p(hit) would be .388. A reduction in sigma to 70 meters increases the probability of hit to the same level as doubling the number of rounds. These observations give guidelines for analysis of the benefits of intelligence versus increasing the number of munitions.

V. GENERAL CASE

Guns do not usually hit the exact spot at which they aim. Errors associated with this can be divided into the two categories of MPI errors and precision errors. (Errors associated with vibration will be discussed in Part II). Sandmeyer (1985) discusses these errors and how they effect the optimal munition density. In the direct fire case both the TLD and MPI errors combine to give a convoluted TLD; thus the magnitude of the MPI errors diffuse our knowledge of the TLD. Intuitively MPI errors can be considered a form of target uncertainty, since the fire control system cannot determine the center of impact in relation to the target before the shots are fired. In effect the target was thought to be at the center of impact but is at a different location. The precision errors spread out the effect of the hit or kill function by distributing it over a larger area; a convolution of the kill function and the precision errors yields a modified kill function that can be used to find an optimal munition density.

Probability of Hit

To determine the probability of hitting the target given its location, both the size of the target and the precision errors are needed. If the location of the target is known and MPI errors are ignored, the probability of hitting the target can be calculated by integrating the precision errors over the area of the target. This is expressed mathematically as follows:

E_T - elevation dimension of the target

A_T - azimuth dimension of the target

$g(A,E)$ - the precision error distribution in azimuth and elevation units

$$\int_{-A_T/2}^{A_T/2} \int_{-E_T/2}^{E_T/2} g(A,E)\, dE\, dA$$

Precision errors are almost exclusively modeled by a bivariate normal; thus, there are many documented methods to calculate their value.

Example 3:

Suppose a target 2.5m by 3.5m is 2km from the shooter. Assume the precision error is 5 mils in both azimuth and elevation. Then what is the probability of hitting the target if the gun is properly aimed? Recall that $\sin \Theta \approx \Theta$ when Θ is small and Θ is measured in radians. Two methods will be shown.

Method One

Solve the problem in meters:

At a range of 2km, 5mils is equal to 10 meters

$$\sin (5 \text{ mils}) \approx .005$$

$$.005 = N/2000 \qquad N=10$$

$$\int_{-1.75}^{1.75}\int_{-1.25}^{1.25} \frac{1}{2\pi 100} e^{\frac{-1}{2}\frac{(A^2+E^2)}{100}} \, dEdA = .0035$$

Method Two

Convert the size of the target to mils and solve

$$\frac{2.5}{2000} = 1.25\text{mils}$$

$$\frac{3.5}{2000} = 1.75\text{mils}$$

$$\int_{-.875}^{.875}\int_{-.625}^{.625} \frac{1}{2}\pi(25)\, e^{\frac{-1}{2}\frac{(A^2+E^2)}{25}} \, dEdA = .0035$$

Effects of a Shot

To calculate the probability of hitting a target for a particular shot, the probability of the shot hitting each position of the TLD should be multipled by the probability that the target area covered that location. For a rectangular target this amounts to integrating the TLD over the area of the target and multiplying this value by the value of the precision function at that point. If this is done for all values of the precision function, then the expected damage for a shot fired at a given point can be calculated.

g(X,Y)	represent the precision errors	X azimuth Y elevation
h(A,E)	represent the TLD with MPI errors included	A azimuth E elevation
(a,b)	aimpoint is azimuth, elevation units	

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{Y-E/2}^{Y+E/2}\int_{X-A/2}^{X+A/2} g(X-a, Y-b)h(A,E)\, dA\, dE\, dxdy$$

If the convoluted TLD is large compared to the size of the target then the average value of the TLD over the target area will be very close to the value at the center of the target rectangle; thus in some instances the above formula can be approximated by

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(X - a, Y - b)h\,(x,y)\,A_T\,E_T\,dxdy$$

A_T is the azimuth dimension of the target.
E_T is the elevation dimension of the target.

VI. PARAMETRIC STUDY PRECISION ERROR

A study was done to observe the effects of the precision errors on the optimal aimpoint pattern and effect. The MPI errors were fixed at 5 mils. and the 2.5 x 3.5 target was assumed to be 2 kilometers away. In each case 60 shots were used. The precision errors were varied to find the point at which it becomes pragmatic to use multiple aimpoints. The investigation was based on a software package developed by Richard Sandmeyer.

Software

A collection of subroutines and functions was developed to find the optimal aimpoint pattern and assess the effects of the pattern. The original version was designed for the Cyber and was modified to run on the Cray. The new version was validated by comparing results to previous results from the Cyber. The software has been documented, but an understanding of the software requires one to understand both the theory of the method, many sophisticated numerical analysis techniques, and methods of computer science. Two types of damage functions are supported by this software: the Von Nuemann-Carleton damage function and a "cookie-cutter" damage function. For a discussion of these see Army Weapons System Analysis, Chapter 20. The Von Nuemann-Carleton damage function was used in this study. An "equivalent" representation of damage was done using the cookie-cutter method and the results were in agreement. The inputs to the model are as follows:

Standard deviation of MPI elevation error - 10m

Standard deviation of MPI azimuth error - 10m

Standard deviation of precision elevation error - varied

Standard deviation of precision azimuth error - varied

Carleton Damage Function Parameters

Ratio of axes	.71
Max value of junction	1.00
Lethal area	8.75

Target Dimensions

Elevation Range	3.5m
Azimuth	2.5m

Seven cases were run, both the azimuth and elevation precision error standard deviations were set to the same value in each case.

The results are displayed graphically. Figure 2 shows the actual effect achieved as a function of precision error. Figure 3 shows the ratio of the achieved effects to a theoretical upperbound as a function of precision error. Figure 4 gives the number of aimpoints as a function of the precision errors. Note that when the precision errors are equal to or greater than the MPI errors there is only one aimpoint, so the best policy is to shoot where you think the target is. This chart also indicates the increase in the number of aimpoints is exponential as the precision errors decrease.

Using the aimpoints for a precision - error of two meters, a monte carlo simulation was used to observe the effectiveness of the pattern. This result was in close agreement with expected result. The monte carlo model was also run with every shot aimed at the center. The use of multiple aimpoints was 2.8 times more effective in hitting the target.

Discussion

The results show that if the precision errors are greater than the MPI errors the best policy is to fire at the center of the target. This results in lower performance as the precision errors increase since the shots become more likely to hit a spot where the target location density has a low value. As the precision error decreases firing all the shots at one location results in overkill for that area. Some of these shots could be used to cover a larger area of the target location density. After the precision errors fell to half the value of the MPI errors, there was a small increase in performance and a large increase in the number of aimpoints. Part II of this paper will investigate the order of firing and different firing policies, investigate the effects of the target at different ranges, and look at different MPI errors.

VII. FUTURE INVESTIGATIONS

The methods and techniques needed to find optimal aimpoint patterns have been discussed and their use has been demonstrated. Other areas that can be investigated are :

a) The effects of target size

b) The effects of changing the MPI errors

c) The effects of vibrational errors

The methods discussed could be used to find the response surface around an operating point. Questions can be answered that address the desirability of reducing precision errors to a certain level and whether the increase in system complexity is justified by the cost increase. Further analysis needs to be done to find the average number of shots till the first hit given a hit occurs for various aimpoint policies.

Multiple aimpoints are detrimental for small targets when the precision errors exceed the MPI errors. Studies can be performed to evaluate the effects of reducing target uncertainty versus improving the precision of the shot based on financial constraints. Part II of this paper will discuss these and other issues for the case of a heliborn gun firing at a helicopter.

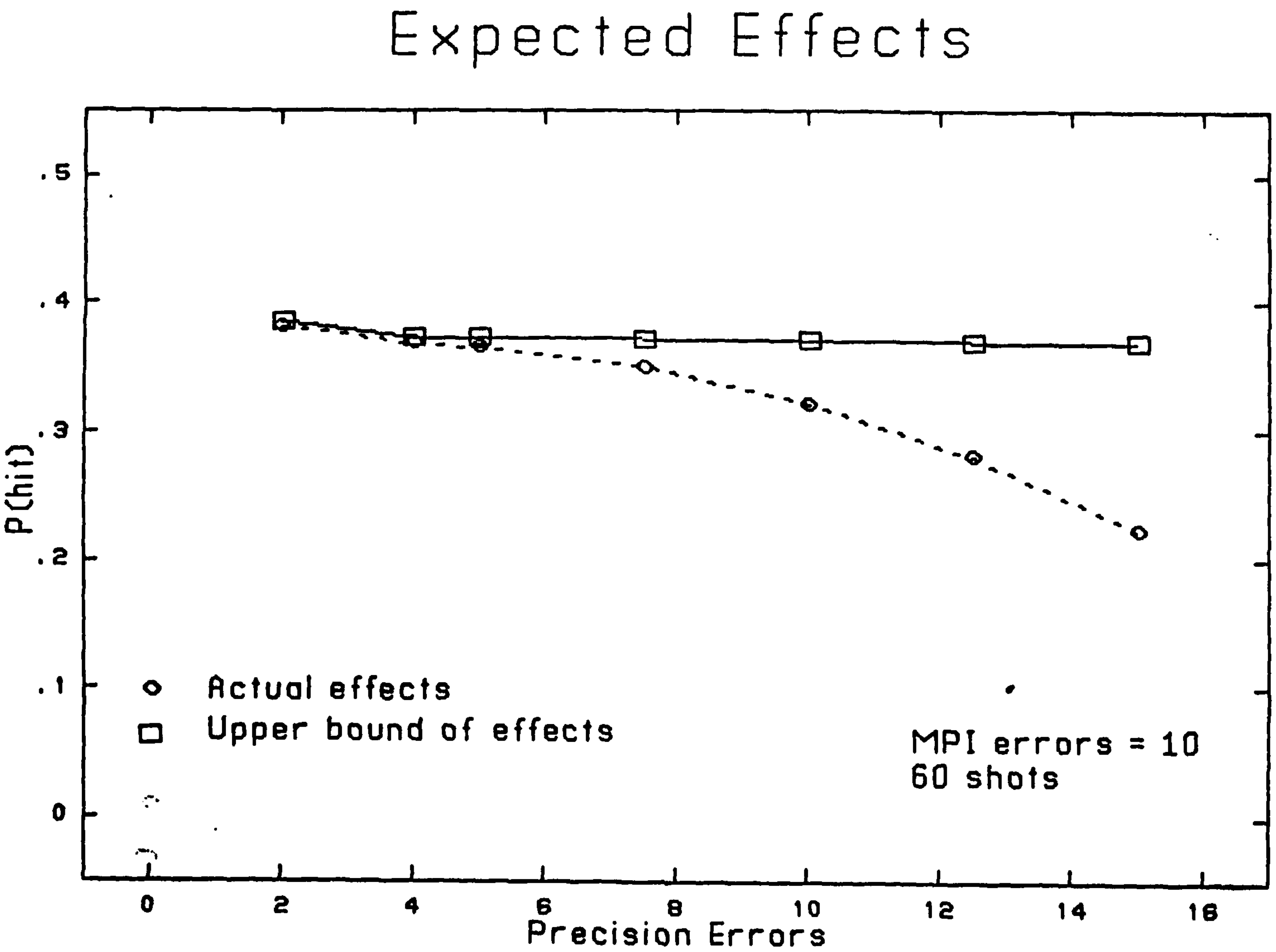

Figure 2. Expected effects.

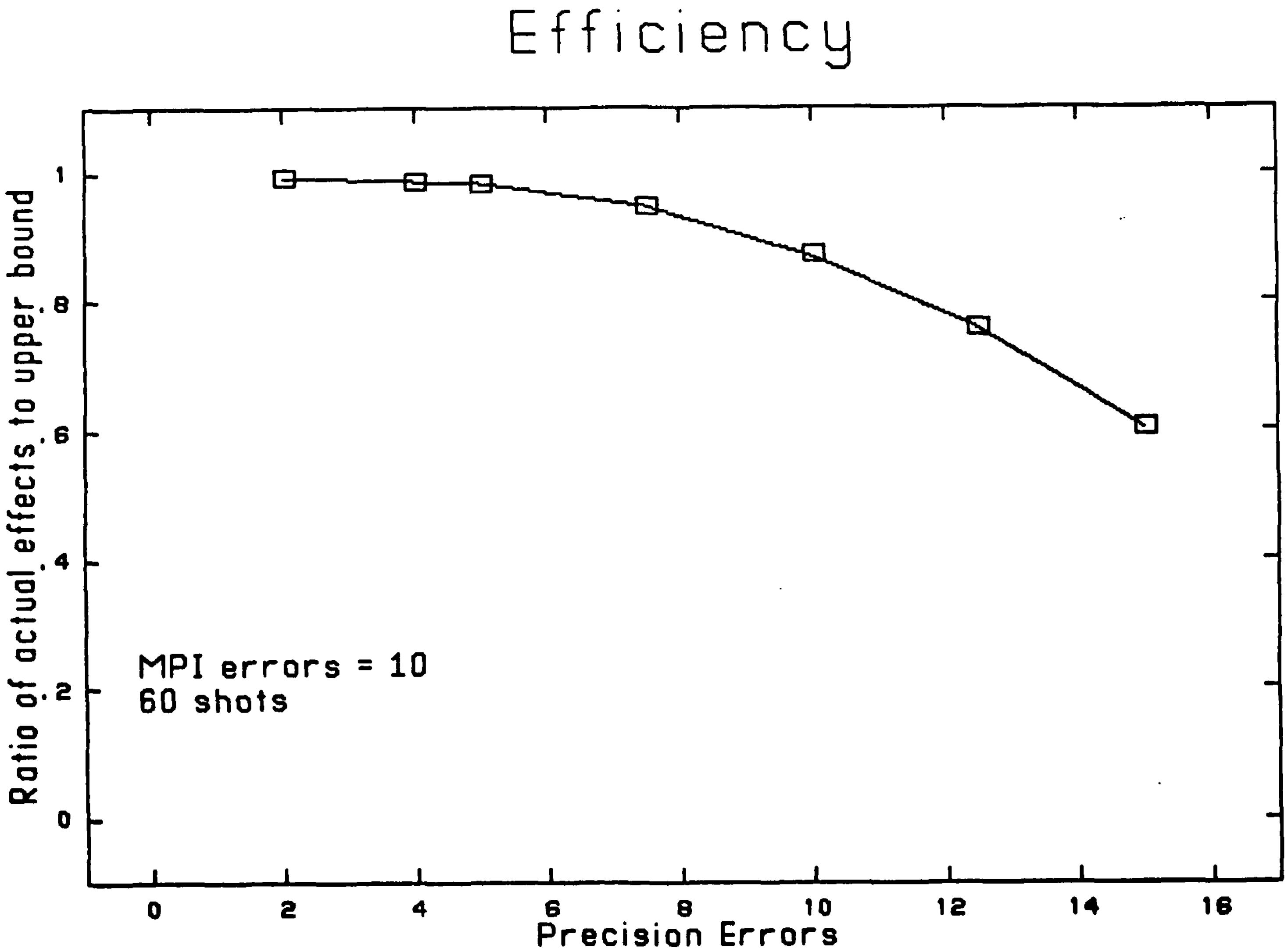

Figure 3. Ratio of actual effect to upperbound.

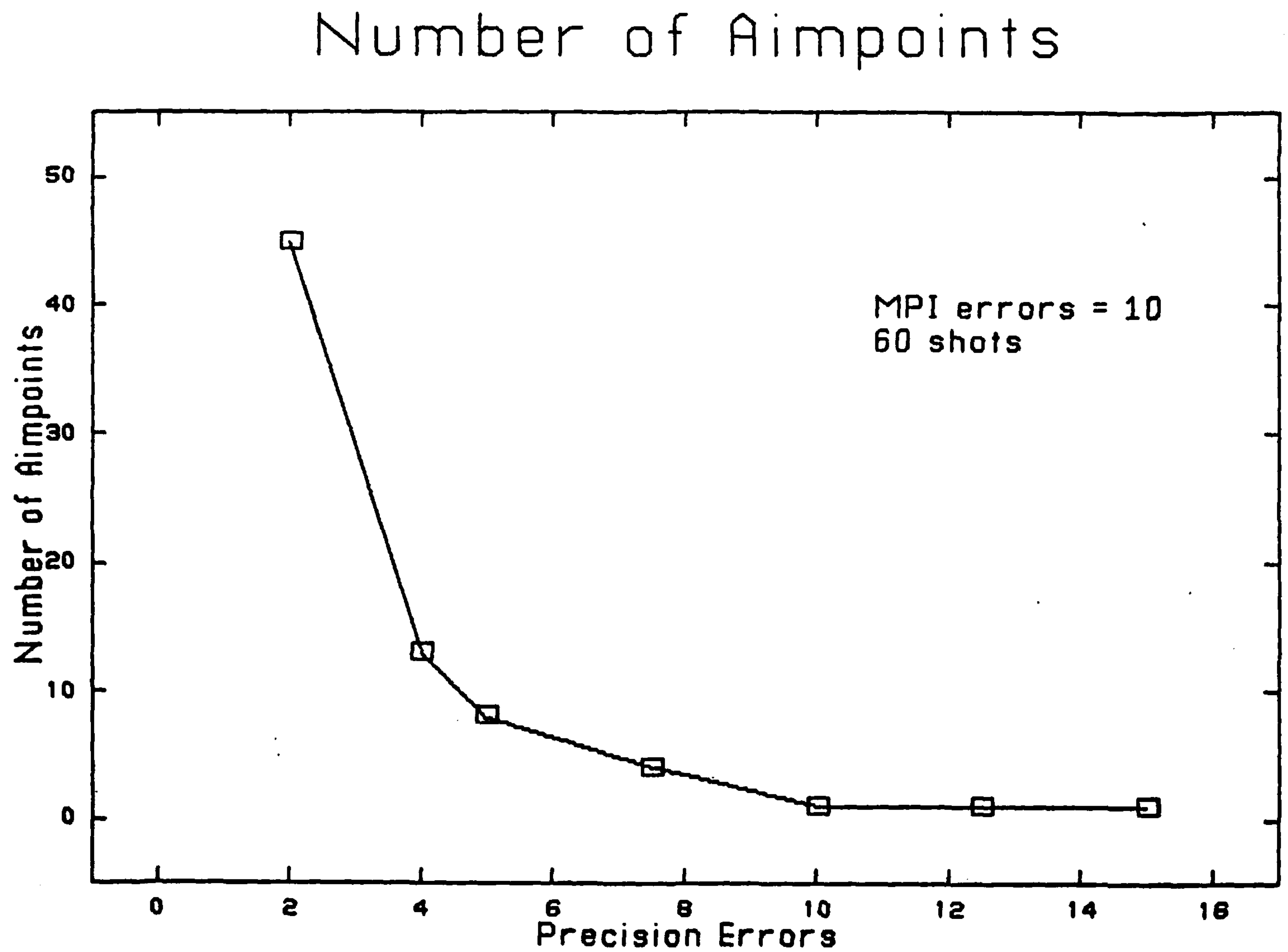

Figure 4. Optimal number of aimpoints as a function of precision error.

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